Magnetic Electron Turbulent Transport.

David W. Ross, August 23, 2001.

Initial thoughts on reading Rechester and Rosenbluth, **PRL** 40, 38 (1978). Take the following with a grain of salt.

The bottom line for me is the question: are the field lines stochastic in the code, i.e., is the island overlap criterion satisfied? If so, I argue that our result can make sense.

To me it looks as if the R and R result has nothing to do with reconnection of the field lines. Is this right?

1. The Rechester-Rosenbluth result

R and R assume that the field lines are stochastic, i.e., an initial bundle of field lines spreads into thin fingers (their figure 1). They assume that the perturbed field can be expanded into Fourier harmonics

$$\delta\mathbf{B} = \sum_{m,n} b_{mn}(r)\exp\left[i(m\theta - nz/R)\right], \tag{1}$$

and define a field line diffusion coefficient

$$D_{st}(r) = \pi R \sum_{m,n} \frac{\left|b_{mn}(r)\right|^2}{B_z^2}\delta\left(\frac{m}{q(r)} - n\right). \tag{2}$$

I presume the latter can be integrated over the box. (They use g to represent q in their Eq. (5).) Then, in the *collisionless limit*, the electron thermal diffusion coefficient is given by

$$\chi_e = D_{st}v_e \tag{3}$$

where v_e is the electron velocity, e.g., the thermal velocity. This result is *independent of the collision frequency or mean free path*, provided there are some small collisions providing a perpendicular step. The point is that the perpendicular step need only be larger than the width of the stochastic filaments, which become exponentially small as the correlation length is exceeded.

Thus, the electron mean free path need only be longer than the correlation length of the perturbed magnetic field, and a step size of order the electron gyro-radius is sufficient. In our case, I think this can be easily satisfied.

In dimensionless units,

$$\hat{\chi}_e \equiv \frac{\chi_e a}{\rho_i^2 v_i} = \frac{\overline{D}_{st} a}{\rho_i^2}\left(\frac{m_i}{m_e}\right)^{1/2}, \tag{4}$$

where, averaged over the box of width $\rho_i \Delta$, with Δ the dimensionless width, and using the dimensionless field perturbations, $\hat{b}_{mn} = (a/\rho_i) b_{mn}/B_z$, we get

$$\begin{aligned}
\frac{\overline{D}_{st} a}{\rho_i^2} &= \frac{\pi R a}{\rho_i^3 \Delta}\int_0^{\rho_i \Delta} dr \sum_{m,n} \frac{|b_{mn}(r)|^2}{B_z^2}\delta\left(\frac{m}{q(r)} - n\right) \\
&= \frac{\pi R}{a\rho_i \Delta}\int_0^{\rho_i \Delta} dr \sum_{m,n} \left|\hat{b}_{mn}(r)\right|^2 \delta\left(\frac{m}{q(r)} - n\right) \\
&\frac{\pi R q}{a\Delta}\sum_{m,n}\frac{1}{(k_\theta \rho_i)_m \hat{s}}\left|\hat{b}_{mn}(r_{mn})\right|^2 .
\end{aligned} \tag{5}$$

where we have noted $q' = \hat{s}q/r$ and $q(r_{mn}) \approx q$. Earlier, I estimated the separations between the rational surfaces r_{mn} in dimensionless units, i.e., r_{mn}/ρ_i. We have to figure out how to add these up to get the final results.

2. The overlap criterion. Are the fields stochastic?

To determine whether the fields are stochastic, we could try to follow field lines. An estimate is from the overlap criterion of Zaslavsky and Chirikov, Sov. Phys. Usp. **14**, 549 (1972), which R and R quote as

$$s = \frac{1}{2}(\Delta_{mn} + \Delta_{m'n'})/|r_{mn} - r_{m'n'}| \tag{6}$$

where the island width is given by Rechester and Stix, PRL **36**,587 (1976). (Rechester and Rosenbluth's Eq. (2) has a couple of typos in it.)

$$\Delta_{mn} = 4\left[2\frac{R}{m}\left|\frac{b_{mn}(r)}{B_z(d\bar{\iota}/dr)}\right|_{r=r_{mn}}\right]^{1/2}, \tag{7}$$

where $\bar{\iota} = 1/q$. In dimensionless form, I find

$$\frac{\Delta_{mn}}{\rho_i} = 4\left(\frac{\hat{b}_{mn} R/a}{(k_\theta \rho_i)_m \hat{s}/q}\right)^{1/2}. \tag{8}$$

Previously, I estimated the rational surface spacing between neighboring *m* values for a fixed *n*. This is probably not the correct value to look at, but for the record it is

$$\frac{\delta_{rat}(m,m+1)}{\rho_i} = 1\Big/\left[\left(k_\theta \rho_i\right)_m \hat{s}\right]. \tag{9}$$

The ratio is

$$\frac{\Delta_{mn}}{\delta_{rat}(m,m+1)} = r\left[\left(\hat{b}_{mn} R/a\right)\left(k_\theta \rho_i\right)_m q\hat{s}\right]^{1/2}. \tag{10}$$

3. Discussion of electron transport.

If the electron boundary conditions have some randomness at the ends of the domain, then the effective mean free path is the domain length for our purposes, regardless of the true collisional mean free path, which is certainly longer. If the parallel correlation length of the field lines is shorter than the length of the domain, then we should get the Rechester-Rosenbluth result, which is correct in this limit and independent of collisionality. What I am ignorant about is how the electron dynamics and boundary conditions are calculated.

In dimensionless units, Eq. (4) shows that the transport decreases as the electron to ion mass ratio increases. I believe this was is the opposite of what I was getting before with the effect limited to leavers close to each rational surface.